STAT 721: Stochastic Processes

Spring 2022

Instructor:	Ray Bai	**Time:**	1:10-2:00 pm MWF
Email:	RBAI@mailbox.sc.edu	**Place:**	Darla Moore School of Business 122

Course Page: https://blackboard.sc.edu/ (Check regularly for updates)

Office Hours: By appointment. I am also very accessible by e-mail and will typically reply to e-mails within one business day of receiving them.

COVID-19 Policies: All classes will be in person. The class will only be moved online temporarily if the instructor is required to isolate/quarantine. Students are *required* to wear face coverings in the classroom and are further encouraged to maintain 3-foot social distancing. For university updates on COVID-19, please see https://sc.edu/safety/coronavirus/

Course Description: Stochastic processes are probabilistic (non-deterministic) systems indexed by time or space. They are very useful for modeling a variety of phenomena that vary in a random manner, such as the volatility of financial assets or the spatial distribution of crimes in a city. Stochastic processes are also very useful for many machine learning tasks such as regression, classification, and clustering.

Whereas a purely theoretical course on stochastic processes would likely focus on the study of stochastic processes as mathematical objects and rigorously prove theorems using measure-theoretic probability theory, this course instead studies stochastic processes as *applied modeling* tools. Theory is introduced in this class, but the theory is meant to motivate the methodology and applications. This class is designed to expose students to a variety of applications of stochastic processes, which can facilitate further self-study if desired. The tentative schedule of topics is:

- **Weeks 1-2**: point processes (Poisson processes, spatial point processes, simulation algorithms)
- **Weeks 3-5**: mathematical finance (pricing of European options, random walks, Brownian motion, stochastic differential equations, binomial model, Black-Scholes model)
- **Weeks 6-8**: Gaussian processes (stationarity vs. nonstationarity, GP regression and classification, global approximation methods for GPs in big data)
- **Weeks 9-10**: reinforcement learning (Markov decision process, dynamic programming, Q-learning, policy gradients)
- **Week 11-12**: Markov chain Monte Carlo (Markov chains, Metropolis-Hastings, Gibbs sampling)
- **Week 12-13**: Dirichlet processes (construction of Dirichlet process, Dirichlet process mixture models for clustering and density estimation)
- **Week 14**: group project presentations
- **If time permits**: advanced MCMC topics (Hamiltonian Monte Carlo, embarrassingly parallel MCMC)

Learning Outcomes:

1. Be able to apply stochastic processes to a variety of tasks, including financial modeling, spatial statistics, regression, classification, and clustering.

2. Be able to implement the models covered in the course and design new algorithms based on algorithm design paradigms such as greedy algorithms, divide-and-conquer, and dynamic programming.
3. Be able to communicate effectively through writing scientific reports and making presentations.

Prerequisites: STAT 512 and MATH 544 or equivalent. Students should also be comfortable with a programming language such as R, Python, MATLAB, or C++.

Main References: We will use typed handouts prepared by the instructor. Parts of these lecture notes are *not* complete and will be filled in during lecture. Thus, it is in your best interest to attend lectures.

Computing: This course involves programming. Please use one of the following languages: Python, R, MATLAB, C, or C++.

Homework: There will be four or five homework assignments. Students will work in small groups, and each group will submit a single, typed report for each homework assignment. The homework may consist of both conceptual/theoretical exercises and questions that involve programming.

All homework reports should be typed, including answers to math exercises and data analysis portions that may include figures, tables, etc. Your code should be e-mailed to the instructor in a Zip file.

Project: Students will work in small groups to research a topic of their choosing, prepare a 15-minute presentation, and write a short report in the style of a journal article: abstract, introduction, method, data analysis, and a bibliography. The last week of the semester will be devoted to project presentations. Some potential examples of projects include:

- finance models and numerical methods for pricing American options
- Hawkes process for modeling social media contagion
- local approximation methods for Gaussian processes
- nested Dirichlet process for learning group level distributions from data
- deep reinforcement learning
- Gillespie algorithm for simulating stochastic equations of molecular reactions
- other applications, including economics, finance, physics, biology, social sciences

Students are strongly encouraged to pursue projects that are relevant to their current research or their research interests. Projects must be approved in advance by the instructor, and no two groups may do the same topic for their project. If you have an idea of what you want to do for your project, please "claim" it early. Detailed instructions for the presentation and the report will be given at a later date.

Grading: Your grade will be determined by homework (70%) and the project (30%). The tentative grading scale is as follows: 90-100 for an A, 80-89 for a B+, 70-79 for a B, 60-69 for a C+, 0-59 for a C.

Honor Code: See the Carolinian Creed in the *Carolina Community: Student Handbook and Policy Guide*. The *minimum* punishment for violations of the USC Honor Code is a grade of zero for the work in question. In accordance with university policy, there may be other punishments, including an automatic F in the class and/or expulsion from the university.

Accommodation: If you need special accommodations for any aspects of the course, please contact me before or during the first week of the semester. Note that reasonable accommodations are available for students with a documented disability. If you have a disability and may need accommodations to fully participate in this class, contact the Office of Student Disability Services by phone (803-777-6142) or e-mail sasds@mailbox.sc.edu. All accommodations must be approved through the Office of Student Disability Services.